EXTENDED m-TRIANGULAR DESIGNS AND THEIR APPLICATION AS THREE FACTOR MATING DESIGNS

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SUMMARY

The triangular association scheme of Bose and Shimamoto [4] has been extended to a five associate class which is known as extended m-triangular association scheme and this is used as three factor mating designs. The construction of E.M.T. designs, method of analysis and their application have been discussed in the paper.

1. INTRODUCTION

Aggarwal [3] [1] [2] used the L_2 (S) triangular, double triangular and modified double triangular designs as diallel crosses for the methods (I), (II), (III) and (IV) of Griffing [5] respectively. In this paper, the triangular association scheme of Bose and Shimamoto [4] has been extended to a five associate class to be known as extended m-triangular association scheme for $v=\frac{mp(p-1)}{2}$ treatments and further used the designs based on this association scheme, to be known as extended m-triangular (E.m.T) designs as three factor mating designs. The model used for these designs is an extension of the model used by Aggarwal [1] for method (II) of Griffing [5] and is derived from the model given by Hinkelmann [6] [7] for three factor mating designs. Raghavarao [8] may be refered to for the statistical terms used here.

EXTENDED M-TRIANGULAR DESIGNS

Define an extended m-triangular association scheme as follows.

Definition 2.1.

Let the $\frac{m\,p\,(p-1)}{2}$ treatments be denoted by the triplets $\alpha\,\beta\,\gamma$, $\alpha<\beta=1,2,\ldots p$, $\gamma=1,2,\ldots m$.

Two treatment $\alpha\beta\gamma$ and $\alpha'\beta'\gamma'$ are

(*i*) first associates if $\alpha=\alpha'$, $\beta=\beta'$, $\gamma\neq\gamma'$;

(*ii*) second associates if either, $\alpha\neq\alpha'$, $\beta=\beta'$, $\gamma=\gamma'$ or $\alpha=\alpha'$, $\beta\neq\beta'$, $\gamma=\gamma'$ or $\beta=\alpha'\,\alpha\neq\beta'$, $\gamma=\gamma'$;

(*iii*) third associates if either $\alpha\neq\alpha'$, $\beta=\beta'$, $\gamma\neq\gamma'$ or

$$\alpha=\alpha',\ \beta\neq\beta',\ \gamma\neq\gamma' \text{ or } \beta=\alpha',\ \alpha\neq\beta',\ \gamma\neq\gamma'\,; \qquad (2.1)$$

(*iv*) fourth associates if $\alpha\neq\alpha'$. $\beta\neq\beta'$, $\gamma=\gamma'$;

(*v*) fifth associates otherwise,

Clearly for this association scheme

$n_1=m-1$, $n_2=2\,(p-2)$, $n_3=2\,(p-2)\,(m-1)$.

$$n_4=\frac{(p-2)\,(p-3)}{2},\ n_5=\frac{(m-1)\,(p-2)\,(p-3)}{2} \qquad \ldots(2.2)$$

The characteristic roots θ_i with their respective multiplicities α_1 ($i=o$, 1, 2, 3, 4, 5), of the NN′ matrix of E.m.T) designs are given by

$$\theta_o=rk, \qquad \alpha_o=1.$$

$$\theta_1=r-\lambda_1+2\,(p-2)\,(\lambda_2-\lambda_3)+\frac{(p-2)\,(p-3)}{2}(\lambda_4-\lambda_5)\,;\ \alpha_1=m-1$$

$$\theta_2=r+(m-1)\,\lambda_1+(p-4)\,\lambda_2+(m-1)\,(p-4)\,\lambda_3-(p-3)\lambda_4-(m-1)\,(p-3)\lambda_5\,;\ \alpha_2=p-1.$$

$$\theta_3=r-\lambda_1+(p-4)\,(\lambda_2-\lambda_3)-(p-3)\,(\lambda_4-\lambda_5),\quad \alpha_3=(m-1)\,(p-1) \qquad \ldots(2.3)$$

$$\theta_4=r+(m-1)\,\lambda_1-2\,\lambda_2-2\,(m-1)\,\lambda_3+\lambda_4+(m-1)\,\lambda_5,$$

$$\alpha_4=\frac{p\,(p-3)}{2}$$

$$\theta_5=r-\lambda_1-2\,(\lambda_2-\lambda_3)+\lambda_4-\lambda_5;\ \alpha_5=\frac{(m-1)\,p\,(p-3)}{2}.$$

The characteristic roots ϕ_i with respective multiplicities α_i ($i=1$, 2, 3, 4, 5) of the C matrix of the connected E.m.T designs are given by

$$\phi_i=r-\frac{\theta_i}{k}. \qquad \ldots(2.4)$$

APPLICATIONS

Consider first the crosses involving p lines. Ignoring the parents and $F_1'^s$ we get $\frac{p(p-1)}{2}$ matings. Consider one offspring from each of these matings and treat them as parent for the second generation. Cross these new parents with another set of m parent. The total number of crosses for the second generation will be $\frac{mp(p-1)}{2}$. These crosses may be treated as the treatments of E.m.T designs.

Let y_{ijkl}, the yield of the plot in the l-th block to which the ijk-th genotype is alloted be given by

$$y_{ijkl}=\mu+t_{ijk}+\beta_l+e_{ijkl}. \qquad \ldots(3.1)$$

$$i<j=1, 2\ldots p,\ k=1, 2,\ldots, m,\ 1=1, 2\ldots b.$$

where μ is the general mean, t_{ijk} is the true effect of the ijk-th genotype, β_l is the effect of the l-th block and e_{ijkl}'s are random errors assumed to be normally and independently distributed with means zero and constant variances σ^2.

Let μ, t_{ijk}'s and β_l be assumed to be fixed but unknown parameters. The set of equations given in (3.1) with these assumptions, is known as the fixed effect model (Scheffe [10].

Let

$$p=(p_{121}, p_{131}\cdots t_{1s1}, p_{231}\cdots p_{2s1}\cdots p_{s-1,\ s,\ 1}\cdots$$
$$p_{12m}, p_{13m}. p_{1sm}\cdots p_{s-1,\ sm})$$

$$p_{i\cdot k}=\sum_j p_{ijk},\ p_{\cdot jk}=\sum_i p_{ijk},\ p_{\cdot\cdot k}=\sum_i\sum_j p_{ijk} \qquad \ldots(3.2)$$

$$p_{i\cdot\cdot}=\sum_j\sum_k p_{ijk},\quad p=t,\ \hat{t},\ Q$$

where $\hat{t}_{ijk}$ is the least square estimate of t_{ijk} and Q_{ijk} is the adjusted treatment total for the ijk-th treatment and is given by

$$Q_{ijk}=T_{ijk}-\frac{B_{ijk}}{k} \qquad \ldots(3.3)$$

were T_{ijk} is the total of the plots to which the ijk-th treatment is applied and B_{ijk} is the total of the blocks in which ijk-th treatment occurs.

Let

$$t_{ijk}=g_i+g_j+h_k+s_{ij}+\delta_{ik}+\delta_{jk}+\theta_{ijk} \tag{3.4}$$

where g_i and g_j are the common genetic contributions of the i-th maternal line and j-th paternal line for the first generation, or simply the g.c.a. effects of the first kind, h_k is the g.c.a effect of the second kind, s_{ij}, δ_{ik}, δ_{jk} are the two factor s.c.a effects with s_{ji}, $=\varepsilon_{ji}$, ε_{ijk} is the three factor s.c.a effect with $\varepsilon_{ijk}=\varepsilon_{jik}$.

Assume that

$$\sum_i g_i=o, \quad \sum_j s_{ij}=oj\neq i, \text{ for all } i,$$

$$\sum_k \delta_{ik}=o, \text{ for all } i, \quad \sum_i \delta_{ik}=o, \text{ for all } k \qquad \ldots(3.5)$$

$$\sum_k \varepsilon_{ijk}=o \text{ for all } i \text{ and } j,$$

$$\sum_j \varepsilon_{ijk}=o, j\neq i, \text{ for all } i$$

Then

$$g_i=\frac{t_{i..}}{m(p-2)}, \qquad h_k=\frac{t_{..k}}{p(p-1)}$$

$$s_{ij}=\frac{t_{ij.}}{m}-\frac{t_{i..}}{m(p-2)}-\frac{t_{j..}}{m(p-2)} \qquad \ldots(3.6)$$

$$\delta_{ik}=\frac{t_{i.k}}{p-2}-\frac{t_{i..}}{m(p-2)}-\frac{t_{..k}}{p(p-2)}$$

$$\varepsilon_{ijk}=t_{ijk}-g_i-g_j-s_{ij}-h_k-\delta_{ik}-\delta_{jk}.$$

Let x_i, y_i, z_{ir}, u_i, w_i be the characteristic vectors corresponding to the roots ϕ_1, ϕ_2, ϕ_3, ϕ_4 and ϕ_5 of the C-matrix of the connected E.m.T design respectively. Then

$$t'x_i=\left[\sum_{k=1}^{i-1} t_{..k}-(i-1)\, t_{..i}\right]\div [i\,(i-1)\, 2p\,(p-1)]^{\frac{1}{2}}$$

$$i=2, 3\ldots m.$$

$$t'y_i=\left[\sum_{j=1}^{i=1} t_{j..}-(i-1)t_{i..}\right]\div[i(i-1)m(p-2)]^{\frac{1}{2}} \quad i=2,3\ldots p. \qquad \ldots(3.7)$$

$$t'z_{ir}=\left[\sum_{k=1}^{r-1}\left(\sum_{\lambda=1}^{i-1} t_{\lambda.k}-(i-1)t_{i.k}\right)-(r-1)\right.$$

$$\left.\left(\sum_{\lambda=1}^{i-1} t_{\lambda.r}-(i-1)t_{i.r}\right)\right]\div[i(i-1)r(r-1)p-2)]^{\frac{1}{2}}$$

$$i=2,3\ldots p, r=2,3..m. \qquad \ldots(3.8)$$

Let

$$\sum_{i=2}^{m} x_i x_i' = A_1, \quad \sum_{i=2}^{p} y_i y_i' = A_2; \quad \sum_{i=2}^{p}\sum_{r=2}^{m} z_{ir} z_{ir}' = A_3$$

$$\sum_{i=1}^{\frac{p(p-3)}{2}} u_i u_i' = A_4, \quad \sum_{i=1}^{\frac{(m-1)p(p-3)}{2}} w_i w_i' = A_5$$

Let $A_6 = \frac{1}{m}E_{mm} \times \left[\frac{I_{p(p-1)}}{2} - \frac{2}{p(p-1)}\frac{E_{p(p-1)}}{2}, \frac{p(p-1)}{2}\right]$

Then it can be varified that

$$A_2+A_4=A_6. \qquad (3.9)$$

and

$$A_5=(I_v-A_1-A_2-A_6-V_4-\frac{1}{v}E_{vv})$$

The least square estimates of the various effects are:

$$\hat{g}_i=\frac{Q_{i..}}{m(p-2)\phi_2}, \quad \hat{h}_k=\frac{Q_{..k}}{p(p-1)\phi_1}$$

$$\hat{S}_{ij}=\frac{1}{\phi_4}\left[\frac{Q_{ij.}}{m}-\frac{Q_{i..}}{m(p-2)}-\frac{Q_{j..}}{m(p-2)}\right]$$

$$\hat{\delta}_{ik}=\frac{1}{\delta_3}\left[\frac{Q_{i.k}}{p-2}-\frac{Q_{i..}}{m(p-2)}-\frac{Q_{..k}}{p(p-2)}\right] \qquad (3.10)$$

$$\hat{\delta}_{ijk}=\frac{1}{\phi_5}\left[Q_{ijk}-\frac{Q_{i.k}}{p-2}-\frac{Q_{j.k}}{p-2}+\frac{Q_{i..}}{p-2}+\frac{Q_{j..}}{p-2}\right.$$

$$\left.-\frac{Q_{ij.}}{m}+\frac{Q_{..k}}{(p-1)(p-2)}\right]$$

4. CONSTRUCTION OF E.m.T DESIGNS,

The constructions of E.m.T. designs are based upon the association matrices $B_0=\frac{I_{p(p-1)}}{2}$, B_1 and B_2 of the triangular association scheme of Bose and Shimamoto [4] with parameters,

$$v^*=\frac{p(p-1)}{2},\ n_1^*=2(p-2),\ n_2^*=\frac{(p-2)(p-3)}{2} \qquad \ldots(4.1)$$

The proofs of the theorems are straightforward and are therefore, omitted.

Theorem 4.1 $N_1=I\otimes(B_0-\bar{B}_1)+E_{mm}(\otimes)\ \bar{B}_1$ where $\bar{B}_1$ is the complement of B_1, is the incidence matrix of an E.m.T design with parameters,

$$v=b=\frac{m(p)(p-1)}{2},\ r=k=\frac{(m-1)(p-2)(p-3)}{2}+m.$$

$$\lambda_1=\frac{(m-2)(p-2)(p-3)}{2}+m,\ \lambda_2=\frac{(m-1)(p-3)(p-4)}{2}$$

$$\lambda_3=\frac{(m-2)(p-3)(p-4)}{2},\ \lambda_4=\frac{(m-1)(p-4)(p-5)}{2}+2(m-1)$$

$$\lambda_5=\frac{(m-2)(n-4)(n-5)}{2}+2(m-1).$$

Theorem 4.2 $N_2=I_m\otimes(B_0-B_2)+E_{mm}\otimes B_2$ is the incidence matrix of an E.m.T design with parameters,

$$v=b=\frac{mp(p-1)}{2},\ r=k=\frac{(m-1)(p-2)(p-3)}{2}+1$$

$$\lambda_1=\frac{(m-2)(p-2)(p-3)}{2},\ \lambda_2=\frac{(m-1)(p-3)(p-4)}{2}$$

$$\lambda_3=\frac{(m-2)(p-3)(p-4)}{2},\ \lambda_4=\frac{(m-1)(p-4)(p-5)}{2}$$

$$\lambda_5=\frac{(m-2)(p-4)(p-5)}{2},\ +2.$$

Theorem 4.3 $N_3=I_m \otimes (B_1-B_2)+E_{mm} \otimes B_2$. is the incidence matrix of an E.m.T design with parrmeters,

$$v=b=\frac{mp(p-1)}{2},\ r=k=\frac{(m-1)(p-2)(p-3)}{2}+2(p-2)$$

$$\lambda_1=\frac{(m-2)(p-2)(p-3)}{2},\ \lambda_2=\frac{(m-1)(p-3)(p-4)}{2}+(p-2)$$

$$\lambda_3=\frac{(m-2)(p-3)(p-4)}{2}+2(p-3),\ \lambda_4=\frac{(m-1)(p-4)(p-5)}{2}+4$$

$$\lambda_5=\frac{(m-2)(p-4)(p-5)}{2}+4(p-4)$$

Theorem 4.4 $N_4=I_m \otimes (B_2-B_0)+E_{mm} \otimes B_0$ is the incidence matrix of an E.m.T design with parameters,

$$v=b=\frac{mp(p-1)}{2},\ r=k=\frac{(p-2)(p-3)}{2}+(m-1)$$

$$\lambda_1=m-2,\ \lambda_2=\frac{(p-3)(p-4)}{2},\ \lambda_3=0,\ \lambda_4=\frac{(p-4)(p-5)}{2},\ \lambda_5=2.$$

Theorem 4.5 $N_5=I_m \otimes (B_2-\bar{B}_1)+E_{mm} \otimes \bar{B}_1$ is the incidence matrix of an E.m.T design with parameters,

$$v=b=\frac{mp(p-1)}{2},\ r=k=\frac{m(p-2)(p-3)}{2}+(m-1)$$

$$\lambda_1=\frac{m(p-2)(p-3)}{2}+(m-2),\ \lambda_2=\frac{m(p-3)(p-4)}{2}$$

$$\lambda_3=\frac{m(p-3)(p-4)}{2},\ \lambda_4=\frac{m(p-4)(p-5)}{2}$$

$$\lambda_5=\frac{m(p-4)(p-5)}{2}+2.$$

Theorem 4.6 If N^* is the incidence matrix of BIB design with parameters,

$v^*=m,\ b^*,\ r^*,\ k^*,\ \lambda^*$, then

$N_6=N^* \otimes B_1$ is the incidence matrix of an E.m.T design with parameters,

$$V=\frac{mp(p-1)}{2},\ b=\frac{b^*p(p-1)}{2},\ r=2r^*(p-2)$$

$$k=2k^*(p-2),\ \lambda_1=2\lambda^*(p-2),\ \lambda_2=r^*(p-2)$$

$$\lambda_3=(p-2)\lambda^*,\ \lambda_4=4r^*,\ \lambda_5=4\lambda^*.$$

Theorem 4.7 If N^* is the incidence matrix of a BIB design with parameters, $v^*=m, b^*, r^*, k^*, \lambda^\alpha$, then

$N_7=N^* \times B_2$ is the incidence matrix of an E.m.T design with parameters.

$$v=\frac{mp(p-1)}{2},\ b=\frac{b^*p(p-1)}{2},\ r=r^*\frac{(p-2)(p-3)}{2}$$

$$k=\frac{k^*(p-2)(p-3)}{2},\ \lambda_1=\frac{\lambda^\alpha(p-2)(p-3)}{2}$$

$$\lambda_2=\frac{r^*(p-3)(p-4)}{2},\ \lambda_3=\frac{\lambda^\alpha(p-3)(p-4)}{2},$$

$$\lambda_4=\frac{r^*(p-4)(p-5)}{2},\ \lambda_5=\frac{\lambda^\alpha(p-4)(p-5)}{2}$$

Theorem 4.8 If N^* is the incidence matrix of a BIB design with parameters,

$v^*=m, b^*, r^*, k^*, \lambda^\alpha$, then

$N_8=N \otimes \bar{B}_1$ is the incidence matrix of an E.m.T design with parameters,

$$v=\frac{mp(p-1)}{2},\ b=\frac{b^*p(p-1)}{2},\ r=r^*\left[\frac{(p-2)(p-3)}{2}+1\right]$$

$$k=k^*\left[\frac{(p-2)(p-3)}{2}+1\right],\ \lambda_1=\lambda^*\left[\frac{(p-2)(p-3)}{2}+1\right]$$

$$\lambda_2=\frac{r^*(p-3)(p-4)}{2},\ \lambda_3=\frac{\lambda^\alpha(p-3)(p-4)}{2}$$

$$\lambda_4=r^*\left[\frac{(p-4)(p-5)}{2}+2\right],\ \lambda_5=\lambda^\alpha\left[\frac{(p-4)(p-5)}{2}+2\right]$$

Theorem 4.9 If N^* is the incidence matrix of a BIB design with parameters

$v^*=m, b^*, r^*, k^*, \lambda^\alpha$ then

$N_9=N \otimes \bar{B}_2$ is the incedence matrix of an E.m.T design with parameters,

$$v=\frac{mp(p-1)}{2},\ b=b^*\frac{p(p-1)}{2},\ r=r^*(2p-3)$$

$$k=k^*[2p-3],\ \lambda_1=\lambda^\alpha(2p-3)$$

$$\lambda_2=r^*p,\ \lambda_3=\lambda^\alpha p,\ \lambda_4=4r^*,\ \lambda_5=4\lambda^\alpha.$$

Example 4.1 Given below is the plan of an E.m.T design with parameters,

$v=30=b$	$r=7=k$	$\lambda_1=3$	$\lambda_2=2$	$\lambda_3=1$	$\lambda_4=0$	$\lambda_5=2$

121	131	141	151	231	241	251	341	351	451
342	242	232	232	142	132	132	122	122	122
352	252	252	242	152	152	142	152	142	132
452	452	352	342	452	352	342	252	242	232
343	243	233	233	143	133	133	123	123	123
353	253	253	243	153	153	143	153	143	133
453	453	353	343	453	353	343	253	243	223
122	132	142	152	232	242	252	342	352	452
341	241	231	231	141	131	131	121	121	121
351	251	251	241	151	151	141	251	241	231
451	451	351	341	451	351	341	123	123	123
343	243	233	233	143	133	133	153	141	131
353	253	253	243	153	153	143	153	143	133
453	453	353	343	453	453	343	253	243	233
123	133	143	153	233	243	253	343	353	453
341	241	231	231	141	131	131	121	121	121
351	251	251	241	151	151	141	151	141	131
451	451	351	341	451	351	341	251	241	231
342	242	232	232	142	132	132	122	122	122
352	252	252	242	152	152	142	152	142	132
452	452	352	342	452	352	342	252	242	232

It may be pointed that there does not exist a BIB design with $v=30$, except the irreducible BIB design. A group divisible design with parameters, $v=30$, $b=25$, $r=5$, $k=6$, $\lambda_1=0$, $\lambda_2=1$ can be obtained from the series

$v=s(s+1)$, $m=s+1$, $n=s$, $b=s^2$, $r=s$, $k=s+1$, $\lambda_1=0$, $\lambda_2=1$ of group divisible designs.

The efficiency of this group divisible design is 0.858 whereas the efficiency of this E.m.T design is 0.873.

5. Construction of E.m.T Designs by the Method of Differences

This method is similar to the method of differences used by Singla [11] for the construction of extended L_2 designs.

Let M be a module of m elements, $0, 1, \ldots m-1$, to each element u of M we associate $\frac{p(p-1)}{2}$ symbols denoted by u_{xy}, $x<y=0, \ldots n-1$ the treatment u_{xy} is said to belong to the class xy. Thus, there will be $\frac{p(p-1)}{2}$ classes in all. Define,

$$u_{xy}-u'_{x'y'}=(u-u')xy\ x'y' \tag{5.1}$$

where $(u-u')$ is an element of M.

Theorem 5.1 Find a set of t initial blocks of size k such that

(*i*) treatment belonging to different classes occur the same number, say r, times in the initial blocks;

(*ii*) treatments in any set are distinct;

(*iii*) the non-zero differenees for which $x=x'$, $y=y'$ occur λ_1 times, the zero differences for which $x\neq x'$,
$y=y'$ or $x=x'$, $y\neq y'$ or $x\neq y'$, $y=x'$

occur λ_2 times, the non-zero differences for which
$x\neq x'$, $y=y'$ or $x=x'$, $y\neq y'$, or $x\neq y'$, $y=x'$

occur λ_3 times the zero differences for which $x\neq x'$, $y\neq y'$ occur λ_4 times, and the non-zero differences for which $x\neq x'$, $y\neq y'$ occur $\lambda_5$times. Then by developing there t initial blocks get an E.m.T design with parameters,

$v=\frac{mp(p-1)}{2}$, $b=mt$, r, k, λ_1, λ_2, λ_3, λ_4, and λ_5.

The following example will explain theorem 5.1

Example 5.1 Let $v=18$, $p=4$, $m=3$, treatments be denoted by the triplets. $\alpha\ \beta\ \gamma$, $\alpha<\beta=0, 1, 2, 3, r=0, 1, 2$.

For the purpose of construction let the treatment $\alpha\beta\gamma$ be written as $\gamma_{\alpha\beta}$ (α, β being suffixes of γ). The following six blocks satisfy the conditions of theorrm 5.1

0_{02}	0_{01}	0_{01}	0_{01}	0_{01}	0_{02}
0_{03}	0_{03}	0_{02}	0_{02}	0_{03}	0_{03}
0_{12}	0_{12}	0_{13}	0_{13}	0_{12}	0_{12}
0_{13}	0_{23}	0_{23}	0_{23}	0_{23}	0_{13}
1_{01}	1_{02}	1_{03}	1_{12}	1_{13}	1_{23}
2_{01}	2_{02}	2_{03}	2_{12}	2_{13}	2_{23}

These six clocks were developed in Mod(3) give an E.m.T design with parameters,

$$v=b=18,\ r=k=6,\ \lambda_1=1,\ \lambda_2=2,\ \lambda_3=2,\ \lambda_4=4,\ \lambda_5=0.$$

The average efficiency factor of this design is 0.868.

It may be noted that a BIB design for $v=18$ does not exist except the irreducible BIB design. Triangular, latin square type and cyclic designs also do not exist for $v=18$. Group divisible designs can be constructed for $v=18$ from the following series given by Raghavarao [8]

$$v=ms,\ b=s^3,\ r=s^2,\ k=m,\ \lambda_1=0,\ \lambda_2=5.$$

From this series we can get two designs with parameters close to parameters of the above E.m.T design, one with parameters, $v=18$, $b=8$, $r=4$, $k=9$, $\lambda_1=0$, $\lambda_2=2$, the other with parameters $v=18$, $b=27$, $r=9$, $k=6$, $\lambda_1=0$, $\lambda_2=3$. The efficiencies of these designs are 0937 and 0.778 respectively. Raghavarao [9] has a given a E.G.D. design with parameters, $v=18$, $b=42$, $r=7$, $k=3$, $\lambda_1=2$, $\lambda_2=0$, $\lambda_3=1$. The efficiency of this design is 0.760. It appears that there is no other design with parameters of the given E.m,T design which is more efficient than this.

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